

(1) Number Properties

- An integer is divisible by:
 - 2 if it is even
 - 3 if the sum of its digits is divisible by 3
 - 4 if it is divisible by 2 twice or if the last two digits are divisible by 4
 - 5 if the integer ends in 0 or 5
 - 6 if it is divisible by both 2 and 3
 - 8 if it is divisible by 2 three times or if the last 3 digits are divisible by 8
 - 9 if the sum of its digits is divisible by 9
 - 10 if it ends in 0
- 2 is the first prime number and the only even prime, 1 is not a prime number
- First 10 primes = 2,3,5,7,11,13,17,19,23,29
- When multiplying integers, if one is even, the result is even
- $O+O=E$; $E+E=E$; $E+O=O$; $O*O=O$; $E*E=E$; $O*E=E$; $O/O=O$; $E/E=O$ or E ; $E/O=E$; $O/E=\text{Non-integer}$
- For evenly spaced sets, median=mean=average of first and last term, sum=mean*number of items in the set
- When counting integers, add one before you are done
 - (Last - First + 1) consecutive integers
 - (Last - First)/Increment + 1 consecutive multiples
- If there are an Odd number of items in a set, the average is an integer
- If there are an Even number of items in a set, the average is not an integer
- The product of (n) consecutive integers is divisible by (n!)
 - The sum of (n) consecutive integers is divisible by (n) if (n) is odd, but it is not divisible by (n) if (n) is even
- Even exponents hide the sign of the base
- Positive numbers between 0 and 1 decrease when raised to higher exponents
- Compound bases raised to higher exponents can be split if multiplying, but not if adding
- In fractional exponents, the numerator is the power, denominator is the root
- Roots only have one solution (not as tricky as exponents)
 - Even roots only have a positive value
 - Odd roots have the same sign as the base
- Roots, like exponents, can be combined when multiplied or divided, but not when added or subtracted
- Order of operations = PEMDAS
- If you add or subtract multiples of an integer, you get another multiple of that integer
 - If you add a multiple of N to a non-multiple of N, the result is a non-multiple of N (same for subtraction)
 - If you add two non-multiples of N, the result could be either a multiple of N or a non-multiple of N
- $\text{GCF} * \text{LCM} = a * b$ (for two numbers, a and b)

- Perfect squares always have an odd number of factors; other integers always have an even number of factors
 - The prime factorization of a perfect square contains only even powers of primes
 - Prime factors of perfect cubes must come in groups of 3
- $N!$ must be divisible by all integers from 1 to N
- When given factored form, distribute; when given distributed, factor
- Use conjugate to simplify a root plus/minus another term in the denominator
 - $a + \sqrt{b}$; conjugate = $a - \sqrt{b}$

(2) Fractions, Decimals, & Percents

- Cross multiplication can be used to compare fractions
- Use smart numbers to solve problems with unspecified amounts
 - Can't use this strategy if there is even one specified amount given
- A percent is a fraction with denominator of 100
- Be careful with the wording on percent change problems
 - Don't forget to add back or subtract the original amount when the percent change is more than 100%
- For date sufficiency problems, all you need to compute percent change is any two of the following: original, change, new
- When raising a fraction to a power, pay attention to both sign and size (relative to +1 and -1)

(3) Equations, Inequalities, & VICs

- Do not assume that the number of equations must be equal to the number of variables
 - With 2 equations and 2 unknowns, linear equations usually lead to one solution (unless the 2 equations are mathematically identical), and nonlinear equations usually lead to 2 (or more) solutions
- Check each solution to absolute value equations by plugging back into the original equation
- Rewrite exponential equations so they have either the same base or same exponent
- When you square an equation to solve it, check the solutions in the original equation
- Beware of disguised quadratics
- Use smart numbers for formulas with unspecified amounts
- Linear sequences are of the form $kn + x$, where k is the difference between successive terms
- If a problem seems to require too much computation, look for a pattern to solve it
- Solve complex function from the inside out
- For direct proportionality problems, set up ratios and cross multiply to solve
 - Set up products (instead of ratios) for inverse proportionality

- For linear growth problems, use $y=mx+b$
- Convert the problem into the units asked for in the answer
- When you multiply or divide an inequality by a negative number of variable, you must flip the sign of the inequality
- Line up multiple inequalities to see whether they can be combined into a compound inequality
- You can add inequalities as long as the signs are facing the same direction
 - Never subtract or divide (you can multiply only if both sides of both inequalities are positive)
- Extreme values are efficient for solving inequalities and equations
- For max/min problems, check the extreme values for each variable
- The smallest square is 0
- Use positive/negative rules to set up scenarios for many inequality problems
- Do not be afraid to use the number line for some absolute value problems
- Remember, $\sqrt{x^2} = |x|$
 - Remember this when taking the square root of an inequality
- Use MADS to manipulate
 - M=Multiplication and division by some number
 - A=Addition and subtraction
 - D=Distributing and factoring
 - S=Squaring and unsquaring
- When you have one variable in more than one absolute value expression, use the algebraic approach
- Multiply or divide two equations when you can cancel a lot of variables in one move
- Use the discriminant of the quadratic formula to determine how many solutions an equation has:
 - If $b^2 - 4ac > 0$, 2 solutions
 - $=0$, 1 solution
 - <0 , no solutions
- n =this term, $n-1$ =previous term, $n+1$ =next term
- Any quantity that doubles in a given amount of time is growing exponentially
- Min/max of quadratics occur when the squared term = 0
- If you must multiply or divide a statement by a variable whose sign is unknown, set up two cases and solve each separately to arrive at two possible scenarios
- You can only square inequalities when both sides have the same sign
 - If both sides are positive, do not flip the inequality
 - If both sides are negative, flip the inequality

(4) Word Translations

- For simple motion problems: $RT=D$ or $RT=W$ (make sure units match)
- When two or more workers work together, their rates add
- To count all your options, multiply the choices for each separate option

- When you have repeated items, divide the total factorial by each repeat factorial to count the different arrangements
- To count possible groups, divide the total factorial by two factorials: one for the chose group, and one for the group not chosen
- If a problem has unusual constraints, try counting total and subtracting the constraints
- For probability, AND means multiply (smaller), OR means add (larger)
 - For the OR formula, remember to subtract the probability that both events occur together
- Sometimes it is easier to calculate the probability that an event will not happen than the probability that the event will happen
- If you know the ratio of the weights, you know the weighted average (and vice versa)
- When using venn diagrams, work from the inside out
- Be careful with rounding on optimization problems
- Focus on extremes to solve scheduling problems
- Draw pictures to conceptualize and avoid traps (there is often a trick!)
- Compute rates in units of work per time, not time per work
- Make sure time units are the same in exponential growth problems
- There are typically more permutations (order matters) than combinations (order does not matter)
 - Combination: $\frac{n!}{[(n-r)!(r!)]}$
 - Permutation: $\frac{n!}{[(n-r)!]}$
- A change to a mean can be computed without knowing all the terms in the set
 - Change in mean = $(\text{new term} - \text{old mean}) / (\text{new number of terms})$
- Use residuals to simplify calculations
 - Residual = data point - mean
 - Sum of residuals = 0

(5) Geometry

- Sum of the interior angles of a polygon = $(n-2)*180$
- Area of a:
 - Triangle = $BH/2$
 - Rectangle = LW
 - Trapezoid = $(B1+B2)H/2$
 - Parallelogram = BH
 - Rhombus = $(D1*D2)/2$
- Surface area = the sum of the areas of all faces
- Volume = LWH
- The sum of the interior angles of a triangle is 180 degrees
- The sum of any two sides of a triangle must be greater than the third side
- Pythagorean theorem: $a^2+b^2=c^2$
- Common right triangles:
 - **3-4-5**, 6-8-10, 9-12-15, 12-16-20

- 5-12-13, 10-24-26
- 8-15-17
- Isosceles right triangle has angle measure of 45-45-90
 - Length of sides are $1 : 1 : \sqrt{2}$ or $x : x : x\sqrt{2}$
 - A square is made up of 2 of these triangles, hence the diagonal of a square = $x\sqrt{2}$
 - The diagonal of a cube = $x\sqrt{3}$
- Equilateral triangles have three equal sides with angles measures of 60 degrees each
- 30, 60, 90 triangles have sides in the following ratios:
 - $1 : \sqrt{3} : 2$ or $x : x\sqrt{3} : 2x$
- If two right triangles have one other angle in common, they are similar
 - If their side lengths are in ratio $a:b$, then their areas will be in the ratio $a^2:b^2$
- The area of an equilateral triangle with side of length $S = [S^2\sqrt{3}]/4$
- For circles, Circumference= πD , Area= πr^2
- The corresponding central angle is half the measure of its inscribed angle
- If one of the sides of an inscribed triangle is the diameter, the triangle is a right triangle
- Surface area of a cylinder = 2 circles + rectangle = $2\pi r^2 + 2\pi r h$
- Volume of a cylinder = $\pi r^2 h$
- There are 180 degrees in a straight line
- Slope = rise/run = $(y_1 - y_2)/(x_1 - x_2)$
- Draw a right triangle to find the distance between two points
- Perpendicular lines have negative reciprocal slopes
- To find the midpoint of a line, find the midpoints of x and y separately
- If you are given perimeter and want to maximize area, use a square
- If you are given two sides of a triangle or parallelogram, you can maximize the area by placing those two sides perpendicular to each other (making them the base and the height)
- The graph of a quadratic is a parabola opening upward or downward, depending on the coefficient of the x^2 term
 - Large coefficient = narrow curve
 - Small coefficient = wide curve
- The number of x -intercepts of a parabola is determined by the sign of the discriminant

(6) Critical Reasoning

- Finding the conclusion is the first step
- Use diagramming to save time, improve comprehension, and focus on argument structure (use the T-diagram)
- Focus on logic, do not get bogged down with details
- Major question types:
 - Find the assumption

- Draw a conclusion
 - Strengthen the conclusion
 - Weaken the conclusion
- Be careful with the wording on EXCEPT questions
- Boundary words limit the scope of an argument and can be useful in identifying incorrect answer choices
- Be sure to read all the answer choices
- Think about extreme examples when you evaluate answer choices with extreme words
- Premises + (**Assumptions**) = Conclusion
 - Note that an assumption must be necessary to the conclusion
- A conclusion that states one particular path is the best way assumes that other paths are not as good
 - Look for an assumption that eliminates an alternate model of causation
- Beware of wrong answer choice patterns:
 - No tie to the conclusion
 - Wrong direction
 - Switching terms
- Be careful with answer choices that contain percents or numbers
- Use the Least Extreme Negation technique
 - Ask yourself whether the conclusion could still be valid even if the answer choice were not true
 - This is helpful when you have two attractive answer choices
- **Draw a conclusion** that does not require any assumptions or information beyond the given premises
- Correct conclusion must be true, according to the information in the premises
- Incorrect answer choices include:
 - Out of scope
 - Wrong direction
 - Switching terms
- To **strengthen an argument**, look for answer choices that:
 - Fixes a weakness of the conclusion
 - Validates an assumption
 - Introduces new supporting evidence
- Use a S-W-Slash chart to keep track of answer choices
- Wrong answer types include:
 - No tie to the conclusion
 - Wrong direction
- To **weaken an argument** look for answer choices that:
 - Exposes a faulty assumption
 - Negatively impacts the conclusion
- The conclusion is the author's position (in problems with argument/counter-argument)

(7) Reading Comprehension

- Identify if the passage is long or short to determine your reading approach
- Focus on the simple story
- Focus early, speed up later
- Components of a passage:
 - The point
 - The background
 - Support
 - Implications
- Use headline lists for short passages
- Read as if you enjoy learning about the subject
- Use the skeletal sketch for long passages (focus on the first paragraph)
- Strategy for reading comprehension questions:
 - For general questions:
 - use a scoring system if stuck between two choices (how many paragraphs does the answer choice relate to)
 - For specific questions:
 - Identify the key words in the question and look for those words in the passage
 - Find proof sentences to defend the answer choice
 - For all reading comprehension questions:
 - Justify every word in the answer choice
 - Avoid extreme words
 - Infer as little as possible
 - Preview the first question
- Question types:
 - General
 - Main idea
 - Organization
 - Tone
 - Specific
 - Lookup
 - Inference
 - Minor types
- Types of wrong answer choices:
 - Out of scope (40-50%)
 - Direct contradiction (20-25%)
 - Mix-up (10-15%)
 - One wrong word (10-15%)
 - True but irrelevant (~10%)
- Always review all answer choices
- Be able to justify every word in the answer choice
- The correct answer will not require any outside knowledge

- Stick closely to the language of the passage; avoid answer choices that go too far beyond that language
- Focus on the simple story, not the details

(8) Sentence Correction

- Split up answer choices
- Concision is important
- The subject and verb must agree
- Parallelism is important (structurally and logically)
- Pronouns must refer to an antecedent in the sentence (it, who/what is it?)
 - The noun being referred to typically should not be in the possessive case
- Noun modifiers should be next to the noun that they modify
 - Usually avoid the possessive case
- Verb modifiers can typically be placed further away from the verb they modify
- Like vs. As
 - Like must be followed by a noun
 - Do not use like for examples, use such as
- Comma vs. semi-colon vs. colon vs. dash
- Between – 2 things, Among – more than 2 things
- Verbs are better than nouns if both are correct
- 3 times as old as, NOT 3 times older than
 - 3 years older than, NOT 3 years as old as