Valuation of Fixed Income Securities

What’s that worth?

Don’t let the math get in the way

Excel is your friend

How could this be hard?

- The coupon is known
- The frequency of the coupon is known
- The maturity is known
- This shouldn't be a big deal

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- But it is…

How could this be hard?

C
(or PMT)

- The coupon is known

n
(or
nper)

- The frequency of the coupon is known
- The maturity is known

FV
(or M)

- This shouldn't be a big deal

- But it is….

The problem is r, or
the interest rate

Bond Pricing is a Time Value of Money exercise

- Apply discounted cash flow →
- To the promised future cash flows
  - Which are a series of coupon payments and the repayment of the full principal at maturity
- The bond price will be the value of all discounted future cash flows
- What rate do we use to discount the cash flows?
  - The market discount rate ("r") is the rate of return required by investors given the risk of the bond

Bond Pricing

$$P = \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \frac{C}{(1+r_3)^3} + \ldots\ldots + \frac{C}{(1+r_n)^n} + \frac{M}{(1+r_n)^n}$$

where:

- P = price
- C = fixed coupon payment
- M = maturity value
- r_1, r_2, r_3.....r_n = discount rates
- n = number of periods

Bond Pricing

PMT

FV

$$P = \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \frac{C}{(1+r_3)^3} + \ldots\ldots + \frac{C}{(1+r_n)^n} + \frac{M}{(1+r_n)^n}$$

PV

where:

- P = price
- C = fixed coupon payment
- M = maturity value
- $r_1, r_2, r_3 \ldots . r_n$ = discount rates
- n = number of periods

The required rate of return

Bond Pricing

- Prices fall as market interest rate rises
  - What would you pay for a bond yielding 4% if the general level of interest rates was 5%?
  - What would you pay for a bond yielding 3% if the general level of interest rates was 2%?
- Interest rate fluctuations are a primary source of bond market **risk** (defined here as **volatility**)
- As it turns out, bonds with longer maturities are more sensitive to fluctuations in interest rates

Relationships

- Bond prices are inversely related to the yield (or required return) = **the inverse effect**
- For the same coupon and time-to-maturity, the percentage price change is greater when rates go down than when rates go up = **convexity effect**
  - This means that bond prices go up easier than they go down
  - It's because the bond still pays a coupon which always has value
- For the same time-to-maturity, a lower coupon bond has a greater price change than a higher coupon bond when their yield's change by the same amount = **coupon effect**
  - Higher coupon payments means investors' returns are a bit more front loaded
- Generally, for the same coupon rate, a longer-term bond has a greater price change than a shorter-term bond when their yields change by the same amount = **maturity effect**

- The price of a fixed-rate bond, relative to par value, depends on the relationship of the coupon rate to the market discount rate.

If the bond price is higher than par value, the bond is said to be traded **at a premium.**	• This happens when the coupon rate is greater than the market discount rate.
If the bond price is lower than par value, the bond is said to be traded **at a discount.**	• This happens when the coupon rate is less than the market discount rate.
If the bond price is equal to par value, the bond is said to be traded **at par.**	• This happens when the coupon rate is equal to the market discount rate.

EXHIBIT 1 Relationships between Bond Prices and Bond Characteristics

				Discount Rates Go Down		Discount Rates Go Up	
Bond	Coupon Rate	Maturity	Price at 20%	Price at 19%	% Change	Price at 21%	% Change
A	10.00%	10	58.075	60.950	4.95%	55.405	–4.60%
B	20.00%	10	100.000	104.339	4.34%	95.946	–4.05%
C	30.00%	10	141.925	147.728	4.09%	136.487	–3.83%
D	10.00%	20	51.304	54.092	5.43%	48.776	–4.93%
E	20.00%	20	100.000	105.101	5.10%	95.343	–4.66%
F	30.00%	20	148.696	156.109	4.99%	141.910	–4.56%
G	10.00%	30	50.211	52.888	5.33%	47.791	–4.82%
H	20.00%	30	100.000	105.235	5.23%	95.254	–4.75%
I	30.00%	30	149.789	157.581	5.20%	142.716	–4.72%

EXHIBIT 2 The Convex Relationship between the Market Discount Rate and the Price of a 10-Year, 10% Annual Coupon Payment Bond

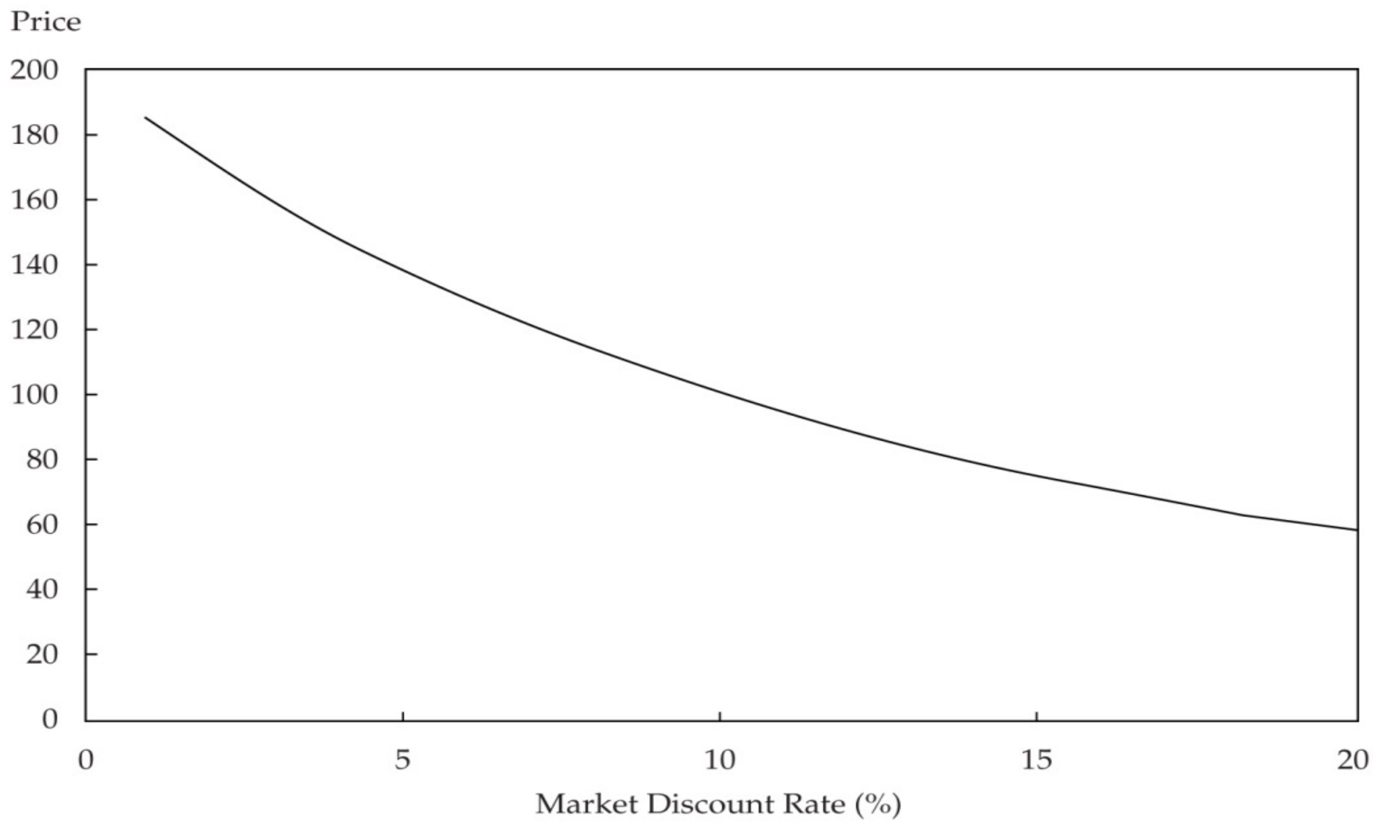
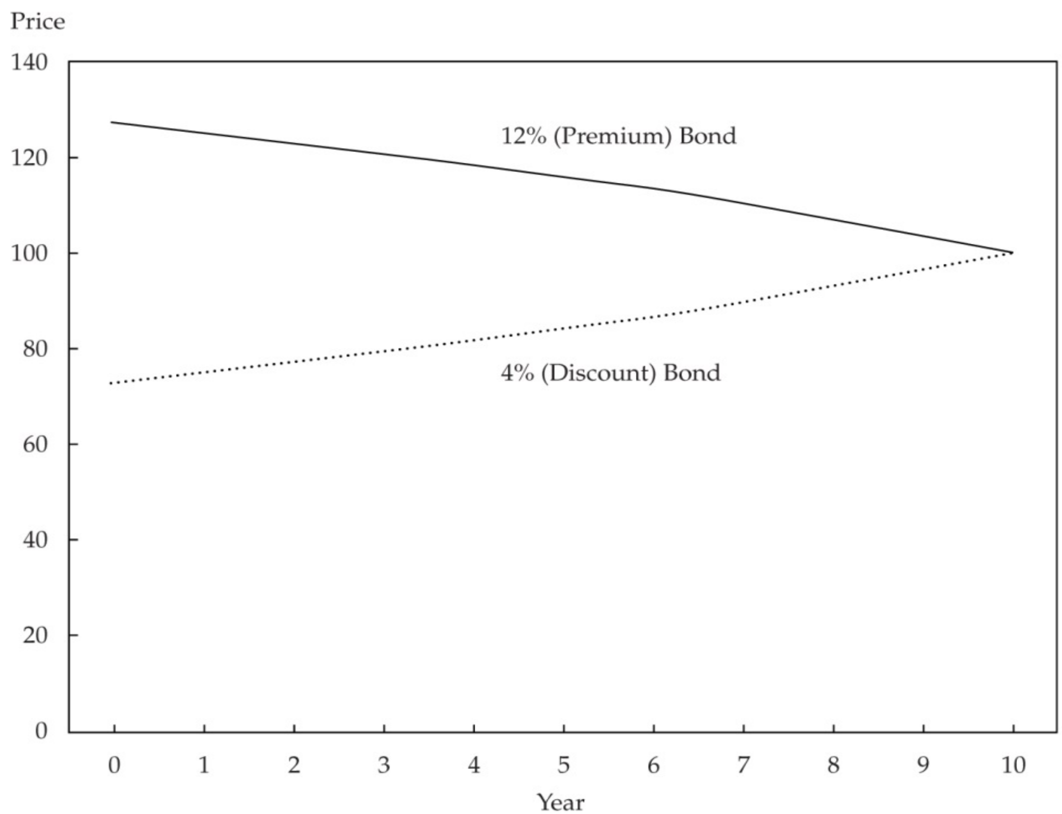


EXHIBIT 3 Constant-Yield Price Trajectories for 4% and 12% Annual Coupon Payment, 10-Year Bonds at a Market Discount Rate of 8%



	0	1	2	3	4	5	6	7	8	9	10
Discount Bond	73.160	75.012	77.013	79.175	81.508	84.029	86.751	89.692	92.867	96.296	100.00
Premium Bond	126.84	124.98	122.98	120.82	118.49	115.97	113.24	110.30	107.13	103.70	100.00

Bond Prices at Different Interest Rates

Time to Maturity	Bond Price at Given Market Interest Rate				
	2%	4%	6%	8%	10%
1 year	$1,059.11	$1,038.83	$1,019.13	$1,000.00	$981.41
10 years	1,541.37	1,327.03	1,148.77	1,000.00	875.38
20 years	1,985.04	1,547.11	1,231.15	1,000.00	828.41
30 years	2,348.65	1,695.22	1,276.76	1,000.00	810.71

Bond Valuation
Interest Accrual

- Bond prices = PV(Coupons) + PV(principal or par value)
- But the inter-coupon interest is owed to the seller (because the buyer will later get the full coupon payment)
- Accrued Interest and quoted bond prices: prices do not included interest that accrues between interest dates
- Accrued interest = (coupon/2) x (days since last coupon payment/days separating coupon payments)
- Excel handles all this well with function =PRICE

Bond Prices: conventions for quotes and calculations

Bond price consists of two components

Flat (clean) price (P_c) + Accrued interest (AI)

The sum of flat price and accrued interest is the full (dirty) price (P_f).

Bond dealers usually quote the flat price.

Buyers pay the full price for the bond on the settlement date.

- Accrued interest is the proportional share of the next coupon payment:

$$\text{AI} = \frac{t}{T} \times \text{PMT}$$

where ***t*** is the number of days from the last coupon payment to the settlement date; ***T*** is the number of days in the coupon period; ***t/T*** is the fraction of the coupon period that has gone by since the last payment; and **PMT** is the coupon payment per period.

30/360 is common for corporate bonds.

The two most common conventions to count days in bond markets:

(*Days in the month/Days in a year*)

Actual/actual is common for government bonds.

Computing Price Using Excel

	6.25% coupon bond,		4.375% coupon bond,		8% coupon bond,
	maturing August 15, 2023	Formula in column B	maturing Nov 15, 2039		30-year maturity
Settlement date	8/15/2011	=DATE(2011,8,15)	8/15/2011		1/1/2000
Maturity date	8/15/2023	=DATE(2023,8,15)	11/15/2039		1/1/2030
Annual coupon rate	0.0625		0.04375		0.08
Yield to maturity	0.02598		0.03697		0.1
Redemption value (% of face value)	100		100		100
Coupon payments per year	2		2		2
Flat price (% of par)	137.444	=PRICE(B4,B5,B6,B7,B8,B9)	111.819		81.071
Days since last coupon	0	=COUPDAYBS(B4,B5,2,1)	92		0
Days in coupon period	184	=COUPDAYS(B4,B5,2,1)	184		182
Accrued interest	0	=(B13/B14)*B6*100/2	1.094		0
Invoice price	137.444	=B12+B15	112.913		81.071

That’s Price, now for Yield

Bond Yields

- Just a few of many many ways to measure yield:
- Yield to Maturity
  - Discount rate that makes present value of bond's payments equal to price.
  - Rate of return if investment held to maturity
- Current Yield
  - Annual coupon divided by bond price
- Holding Period Return
  - Rate of return over particular investment period = f(market price at time of sale)

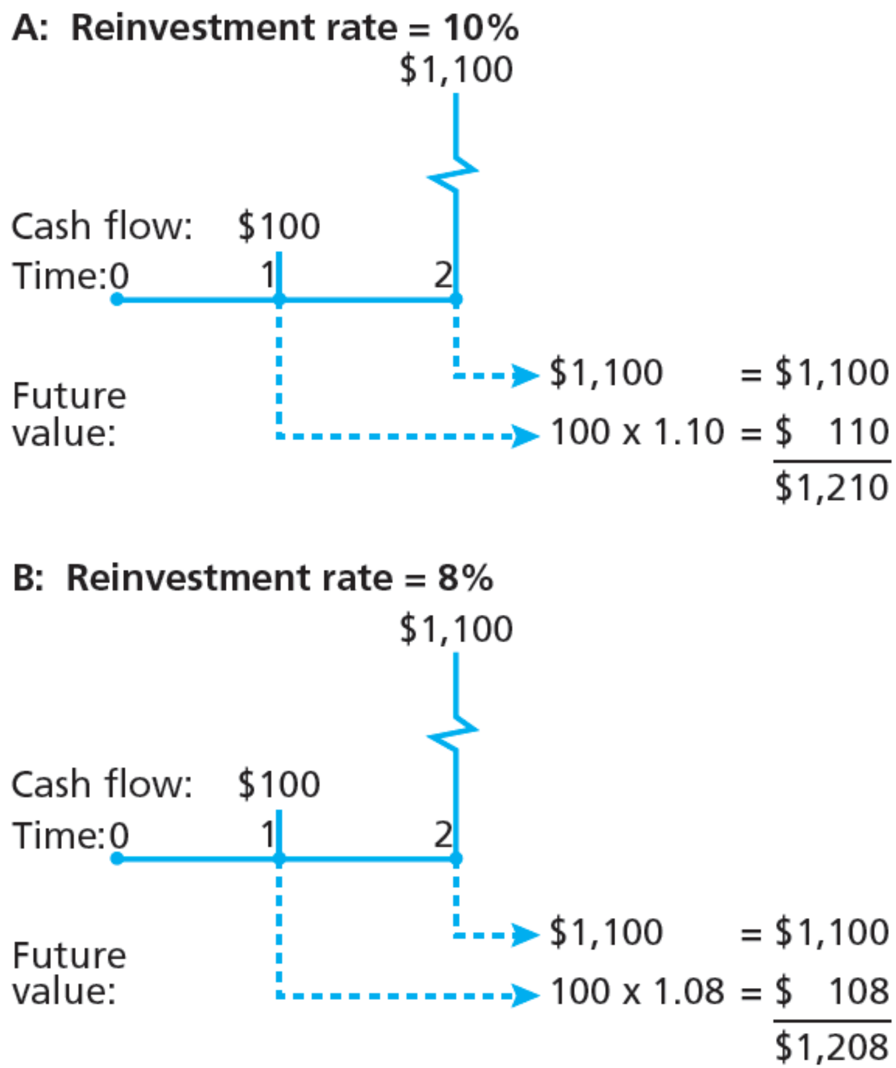
Yield to Maturity

- This is an Internal Rate of Return Concept
- This is the rate of return to the bond investor provided:
  - The investor holds the bond to maturity
  - The issuer does not default on any coupon or maturity payment
  - The investor is able to reinvest coupon payments at that same yield
- So, it’s a promised yield

Computing Yield to Maturity Using Excel

	Semiannual coupons		Annual coupons
Settlement date	1/1/2000		1/2/2000
Maturity date	1/1/2030		1/2/2030
Annual coupon rate	0.08		0.08
Bond price (flat)	127.676		127.676
Redemption value (% of face value)	100		100
Coupon payments per year	2		1
Yield to maturity (decimal)	0.0600		0.0599
The formula entered here is =YIELD(B3,B4,B5,B6,B7,B8)			

Growth of Invested Funds



Excel Exercises

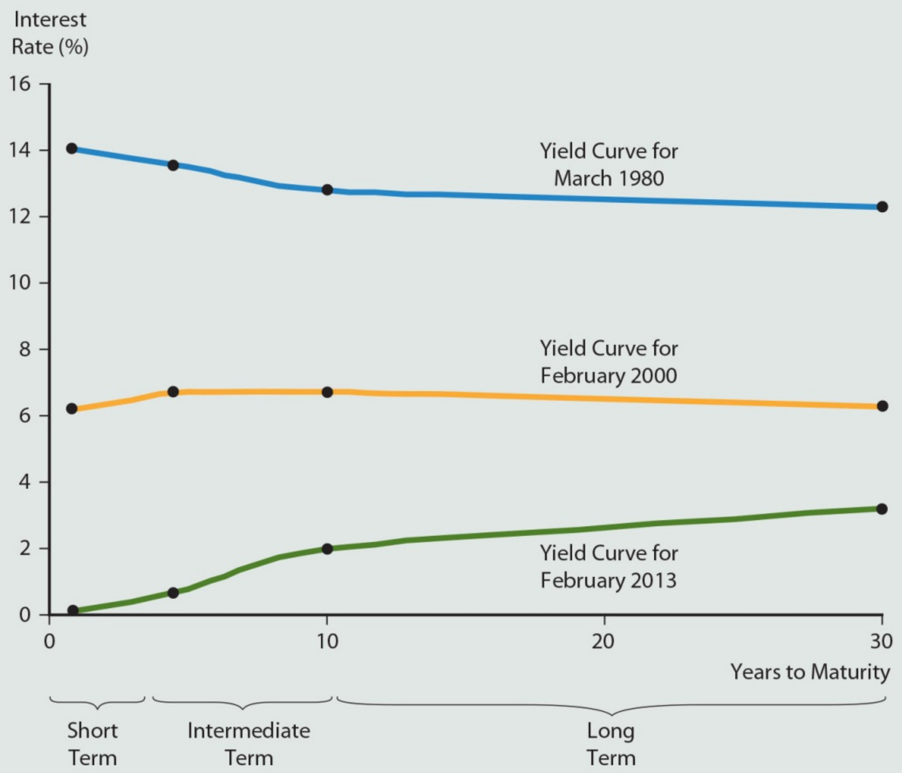
Prices, Yields, and the Yield Curve

- What helps us understand the differences in yields?
- The Yield Curve, it graphs:
  - Time & yields to maturity (on an X-Y axis)
  - aka the “**Term Structure of Interest Rates**”
- The Yield Curve on coupon bonds is a sequence of yields to maturity

Prices, Yields, and the Yield Curve

- Concept of “Interest Rate Risk”
  - Two pieces: a real interest rate and a component to pay for price changes (i.e. inflation)
  - The real interest rate includes:
    - “Expectations”- yields are determined by expectations of future interest rates, and
    - Term (or liquidity) premium – yields reflect demands by investors for compensation for time, extra compensation is required for the greater risk of longer-term bonds
  - Changes in price levels – yields reflect expectations about future changes in inflation/deflation

FIGURE 6.4 U.S. Treasury Bond Interest Rates on Different Dates

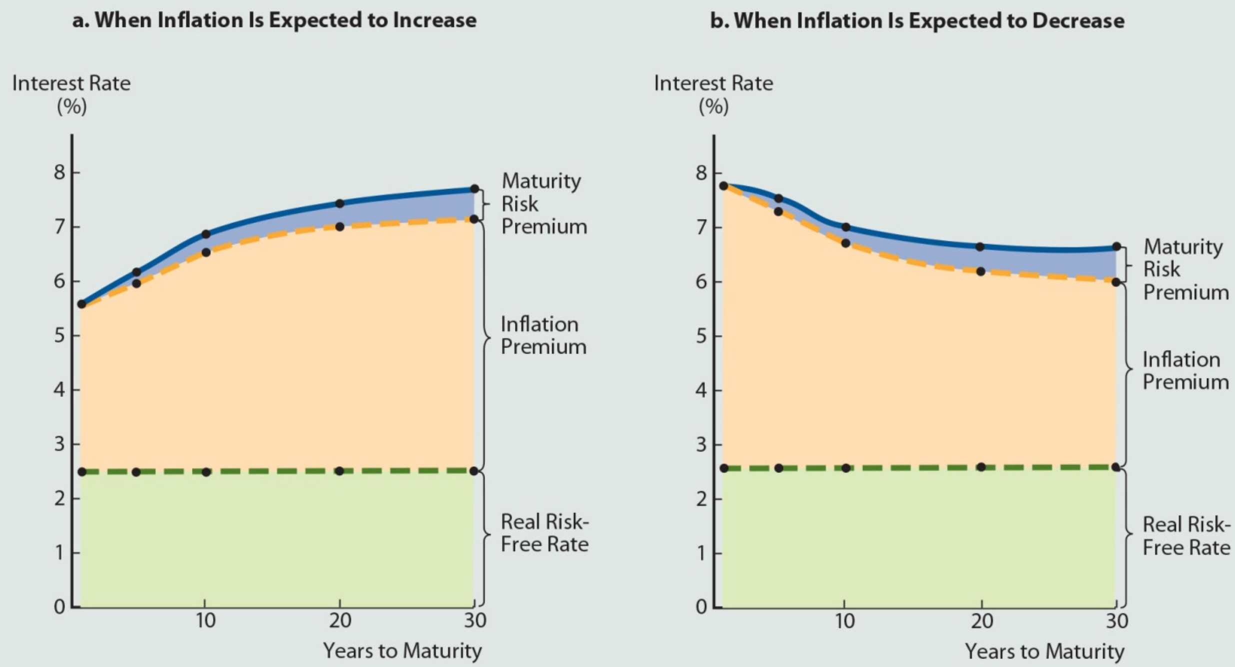


	INTEREST RATE		
Term to Maturity	**March 1980**	**February 2000**	**February 2013**
1 year	14.0%	6.2%	0.1%
5 years	13.5	6.7	0.8
10 years	12.8	6.7	2.0
30 years	12.3	6.3	3.2

Yield Curve Through Time

- http://www.nytimes.com/interactive/2015/03/19/upshot/3d-yield-curve-economic-growth.html?smid=fb-nytimes&smtyp=cur&bicmp=AD&bicmlukp=WT.mc_id&bicmst=1409232722000&bicmet=1419773522000&abt=0002&abg=0&_r=0

FIGURE 6.5 Illustrative Treasury Yield Curves



	With Inflation Expected to Increase					With Inflation Expected to Decrease			
Maturity	**r***	**IP**	**MRP**	**Yield**	**Maturity**	**r***	**IP**	**MRP**	**Yield**
1 year	2.50%	3.00%	0.00%	5.50%	1 year	2.50%	5.00%	0.00%	7.50%
5 years	2.50	3.40	0.18	6.08	5 years	2.50	4.60	0.18	7.28
10 years	2.50	4.00	0.28	6.78	10 years	2.50	4.00	0.28	6.78
20 years	2.50	4.50	0.42	7.42	20 years	2.50	3.50	0.42	6.42
30 years	2.50	4.67	0.53	7.70	30 years	2.50	3.33	0.53	6.36

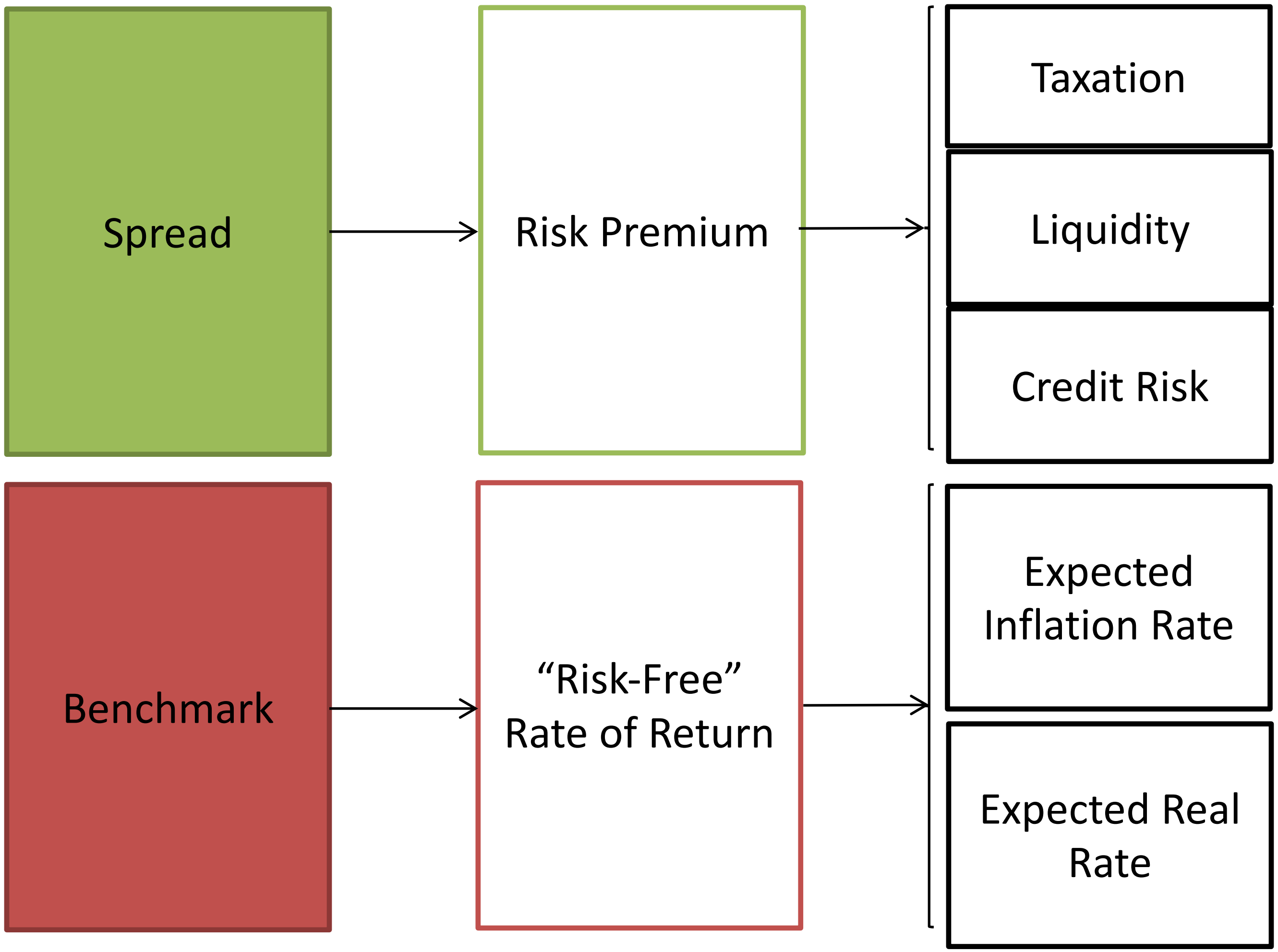
Yield Spreads

The **spread** is the difference between the yield-to-maturity and the benchmark.

The **benchmark** is often called the “risk-free rate of return.” Fixed-rate bonds often use a government benchmark (**on-the-run**) security with the same time-to-maturity as, or the closest time-to-maturity to, the specified bond.

A frequently used benchmark for floating-rate notes is **Libor**. As a composite interbank rate, it is not a risk-free rate.

Yield-to-Maturity Building Blocks



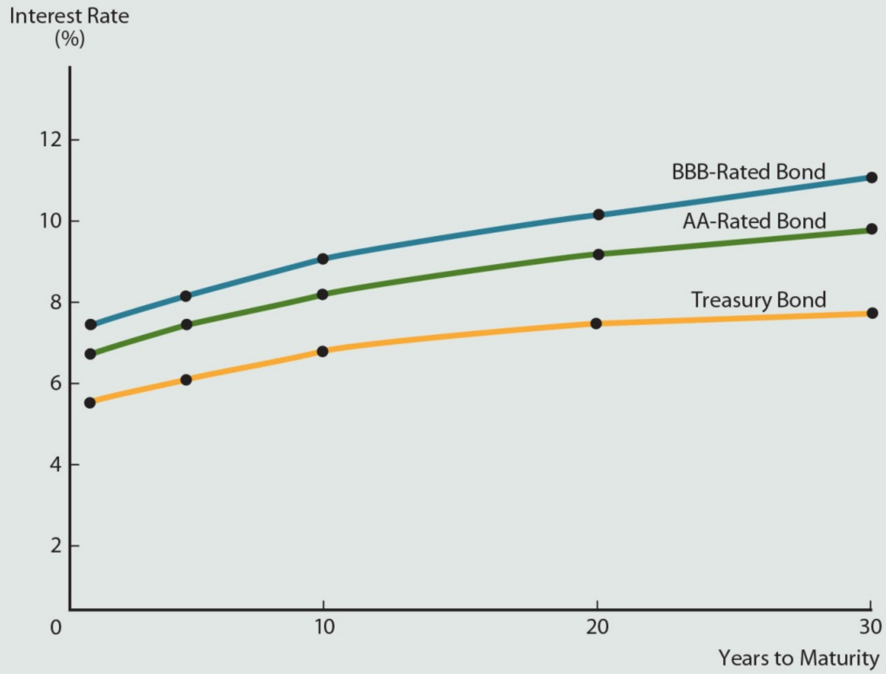
Spreads

- Answers the question: how much additional yield am I receiving/paying vs. US Treasuries?
  - How do we compare across different types of instruments? Fixed, Floating, with options
- G-Spread
  - The difference in yield in **basis points** over an actual or implied government bond
  - Problem if there is a maturity mismatch between appropriate Treasury and the bond we are measuring
- I-Spread
  - The difference in yield in basis points over the standard swap rate (an interbank lending rate, LIBOR being a good example)
  - (We'll get back to the important details around this later in the term)
- OAS – Option Adjusted Spread
  - Adjusted to take into account an embedded option, which is typically taking into account prepayment risk

Swap Rates

- Goal: having a common foundation
  - US Treasuries serves this purpose in the US
- Globally: use LIBOR which is the interbank lending benchmark
- A Swap Rate is the fixed rate that the receiver demands in exchange for paying the uncertain floating LIBOR rate
- From Swap Rates, we get a Swap Curve which is just swap rates at varying maturities
- I Spread works off this common base

FIGURE 6.6 Illustrative Corporate and Treasury Yield Curves



	Interest Rate		
	Treasury Bond	**AA-Rated Bond**	**BBB-Rated Bond**
1 year	5.5%	6.7%	7.4%
5 years	6.1	7.4	8.1
10 years	6.8	8.2	9.1
20 years	7.4	9.2	10.2
30 years	7.7	9.8	11.1

A portfolio manager is considering the purchase of a bond with a 5.5% coupon rate that pays interest annually and matures in three years. If the required rate of return on the bond is 5%, the price of the bond per 100 of par value is *closest* to:

$$PV = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \frac{PMT + FV}{(1+r)^3}$$

where:

PV = present value, or the price of the bond
PMT = coupon payment per period
FV = future value paid at maturity, or the par value of the bond
r = market discount rate, or required rate of return per period

$$PV = \frac{5.5}{(1+0.05)^1} + \frac{5.5}{(1+0.05)^2} + \frac{5.5 + 100}{(1+0.05)^3}$$

$$PV = 5.24 + 4.99 + 91.13 = 101.36$$

A bond with two years remaining until maturity offers a 3% coupon rate with interest paid annually. At a market discount rate of 4%, the price of this bond per 100 of par value is *closest* to:

$$PV = \frac{PMT}{(1 + r)^1} + \frac{PMT + FV}{(1 + r)^2}$$

where:

PV = present value, or the price of the bond
PMT = coupon payment per period
FV = future value paid at maturity, or the par value of the bond
r = market discount rate, or required rate of return per period

$$PV = \frac{3}{(1 + 0.04)^1} + \frac{3 + 100}{(1 + 0.04)^2} = 2.88 + 95.23 = 98.11$$

An investor who owns a bond with a 9% coupon rate that pays interest semiannually and matures in three years is considering its sale. If the required rate of return on the bond is 11%, the price of the bond per 100 of par value is *closest* to:

$$PV = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \frac{PMT}{(1+r)^3} + \frac{PMT}{(1+r)^4} + \frac{PMT}{(1+r)^5} + \frac{PMT + FV}{(1+r)^6}$$

where:

PV = present value, or the price of the bond
PMT = coupon payment per period
FV = future value paid at maturity, or the par value of the bond
r = market discount rate, or required rate of return per period

$$PV = \frac{4.5}{(1+0.055)^1} + \frac{4.5}{(1+0.055)^2} + \frac{4.5}{(1+0.055)^3} + \frac{4.5}{(1+0.055)^4} + \frac{4.5}{(1+0.055)^5} + \frac{4.5+100}{(1+0.055)^6}$$

$PV = 4.27 + 4.04 + 3.83 + 3.63 + 3.44 + 75.79 = 95.00$

A zero-coupon bond matures in 15 years. At a market discount rate of 4.5% per year and assuming annual compounding, the price of the bond per 100 of par value is *closest* to:

$$PV = \frac{100}{(1+r)^N}$$

where:

PV = present value, or the price of the bond
r = market discount rate, or required rate of return per period
N = number of evenly spaced periods to maturity

$$PV = \frac{100}{(1+0.045)^{15}}$$

$$PV = 51.67$$

The following information relates to Questions 8 and 9

Bond	Price	Coupon Rate	Time-to-Maturity
A	101.886	5%	2 years
B	100.000	6%	2 years
C	97.327	5%	3 years

8. Which bond offers the lowest yield-to-maturity?
   A. Bond A
   B. Bond B
   C. Bond C
9. Which bond will *most likely* experience the smallest percent change in price if the market discount rates for all three bonds increase by 100 basis points (bps)?
   A. Bond A
   B. Bond B
   C. Bond C

8. A is correct. Bond A offers the lowest yield-to-maturity. When a bond is priced at a premium above par value the yield-to-maturity (YTM), or market discount rate is less than the coupon rate. Bond A is priced at a premium, so its YTM is below its 5% coupon rate. Bond B is priced at par value so its YTM is equal to its 6% coupon rate. Bond C is priced at a discount below par value, so its YTM is above its 5% coupon rate.
9. B is correct. Bond B will most likely experience the smallest percent change in price if market discount rates increase by 100 basis points (bps). A higher-coupon bond has a smaller percentage price change than a lower-coupon bond when their market discount rates change by the same amount (the coupon effect). Also, a shorter-term bond generally has a smaller percentage price change than a longer-term bond when their market discount rates change by the same amount (the maturity effect). Bond B will experience a smaller percent change in price than Bond A because of the coupon effect. Bond B will also experience a smaller percent change in price than Bond C because of the coupon effect and the maturity effect.

Bond	Coupon Rate	Maturity (years)
A	6%	10
B	6%	5
C	8%	5

All three bonds are currently trading at par value.

11. Relative to Bond C, for a 200 basis point decrease in the required rate of return, Bond B will *most likely* exhibit a(n):
    A. equal percentage price change.
    B. greater percentage price change.
    C. smaller percentage price change.
12. Which bond will *most likely* experience the greatest percentage change in price if the market discount rates for all three bonds increase by 100 bps?
    A. Bond A
    B. Bond B
    C. Bond C

11. B is correct. Generally, for two bonds with the same time-to-maturity, a lower coupon bond will experience a greater percentage price change than a higher coupon bond when their market discount rates change by the same amount. Bond B and Bond C have the same time-to-maturity (5 years); however, Bond B offers a lower coupon rate. Therefore, Bond B will likely experience a greater percentage change in price in comparison to Bond C.
12. A is correct. Bond A will likely experience the greatest percent change in price due to the coupon effect and the maturity effect. For two bonds with the same time-to-maturity, a lower-coupon bond has a greater percentage price change than a higher-coupon bond when their market discount rates change by the same amount. Generally, for the same coupon rate, a longer-term bond has a greater percentage price change than a shorter-term bond when their market discount rates change by the same amount. Relative to Bond C, Bond A and Bond B both offer the same lower coupon rate of 6%; however, Bond A has a longer time-to-maturity than Bond B. Therefore, Bond A will likely experience the greater percentage change in price if the market discount rates for all three bonds increase by 100 bps.

Summary

- Bond's price given a market discount rate
  - The market discount rate is the rate of return required by investors, given the risk of the investment in the bond
  - A bond is priced at a premium above par when the coupon rate is greater than the market discount rate
  - A bond is priced at a discount below par, when the coupon rate is less than the market discount rate
- Relationships among a bond's price, coupon rate, maturity, and market discount rate
  - A bond's price moves inversely with its market discount rate
  - The price of a lower coupon bond is more volatile than the price of a higher coupon bond, other things being equal
  - Generally, the price of a longer term bond is more volatile than the price of a shorter term bond, other things being equal

Summary

- Flat price, accrued interest, and full (dirty) price
  - Between coupon dates, the full (or invoice, or dirty) price of a bond is split between the flat (or quoted, or clean) price and the accrued interest
  - Accrued interest is calculated as a proportional share of the next coupon payment using a particular day count methodology
- Yield measures
  - Understand the difference between YTM, current yield, and realized return

Summary

- Yield Curve
  - Graphed relationship between time and a series of YTMs
- Yield Spread
  - A bond's YTM can be separated into a benchmark yield and a spread
  - Benchmark rates are YTMs of government bonds, or interbank lending rates
  - Changes in spread typically capture microeconomic factors that affect the particular bond: credit risk, liquidity, tax, etc.

Next

- Chapter 3 had more intricate yield calcs for other security types – it’s a bit too detailed for where we want to go here
  - The important take-aways are those things that we did go over
- Assignment: Understand Petitt Chapter 4: Fixed Income Risk and Return
  - Don’t get lost in the weeds of duration calculations
  - How do a bond’s maturity, coupon, embedded options, and yield level effect interest rate risk?
  - Estimate the percentage price change of a bond for a specified change in yield, given duration and convexity
  - Why is effective duration the most important measure of interest rate risk?
  - Understand holding period return and investment horizon
  - Useful chapter end questions are 1-3, 10, 11, 27