31. You have a deck of cards in which aces are excluded. What is the probability of taking the first two cards and both of them being six?

In the first draw I can draw any of the four sixes out of total 48 cards, and on the second draw there are only three sixes out of the remaining 47 cards, so the probability is $\frac{4}{48}\frac{3}{47} \approx 0.005$.

32. (CHECK) You have a deck of cards. What is the expected number of cards you have to draw to get cards of all four suites?

Solution

33. If you have a deck of cards and start putting them in another stack, how many cards do you have to put down to guarantee a Three of a Kind?

Before drawing the card that will guarantee a Three of a Kind, the deck must include all the numbers from 1 to 13 exactly twice, which is total 26 cards. Therefore we will necessarily have a Three of a Kind in a deck of 27 cards.

34. You are playing Russian Roulette. In the chamber there are four blanks and two bullets. If someone shoots a blank next to you, would you spin or take another shot?

After one blank shot, there are two bullets left and three blank positions, if the revolver is not spun again. Therefore the probability that the next shot results in a "brainstorm" is 2/5. Spinning again, the probability is just 1/3, so it's definitely a good idea to spin again.

35. You have a square, and place three dots along the four edges at random. A) What is the probability that the dots, when connected, do not form a triangle? B) What is the probability that the dots lie on distinct edges?

A)
The only way a triange cannot be formed is if all the dots lie on the same edge. The probability of this occurring is

$$P(\text{all 3 on the same edge}) = 1 \cdot \frac{1}{4}\frac{1}{4} = \frac{1}{16},$$

since the first dot can be on any edge.

B)
The first dot can be on any edge, so the second one can be only on the remaining three edges and the last one at the remaining two edges. The probability is thus $1 \cdot \frac{4}{4}\frac{2}{4} = 3/8$.

36. You have three pancakes. One is with both sides burned, one is with one side burned, the last one with no sides burned. You combined them in a plate, and the top side is burned. What is the probability of the other side is burned?

We can denote the pancakes as $A = (1, 1)$, $B = (1, 0)$ and $C = (0, 0)$ where 1 denotes a burned side and 0 a non-burned side. Also denote by 'tsb' that the top side is burned. We are asking for the probability of choosing A given that the other side is observed to be burned. By the Bayes theorem,

$$P(A|\text{tsb}) = P(\text{tsb}|A)\frac{P(A)}{P(\text{tsb})} = \frac{P(\text{tsb}|A)P(A)}{P(\text{tsb}|A)P(A) + P(\text{tsb}|\overline{A})P(\overline{A})}.$$

We have $P(\text{tsb}|A) = 1$, since both sides of A are burned, $P(A) = 1/3 = 1 - P(\overline{A})$, since there are three pancakes to choose from, and $P(tsb|\overline{A}) = 1/4$, since that is the probability we pick B *and* that it is placed burned side up. Therefore,

$$P(A|\text{tsb}) = \frac{1/3}{\frac{1}{3} + \frac{1}{4}\frac{2}{3}} = \frac{2}{3}.$$

37. If there is a thirty percent chance of rain on Monday and a thirty percent chance of rain on Tuesday, what is the probability that it rains at least once during the first two days of the week?

The probability that it will rain at least once is equal to one minus the probability that it will not rain on either day, so the total probability is

$$1 - (\frac{7}{10})^2 = \frac{51}{100}.$$

38. A survey is given to all passengers on a number of different planes. The survey asks each person how full their plane was. If fifty percent of people claim that their plane was eighty percent full, while the other fifty percent claim that their plane was twenty percent full, how full was the average plane?

Note that the solution is not $(80\% + 20\%)2 = 50\%$, since the planes that are 80% and 20% full have different numbers of passengers in them. Suppose then that there are two kinds of planes with 100 seats. If a person says that his plane was 80% full, then that plane had 80 passengers in it. If a person says that her plane was 20% full, then that plane had 20 passengers in it. To get a 50/50 response rate of 80% and 20%, we need to compare an equal number of people, i.e. one plane of 80 passengers and four planes of 20 passengers and compute the average out of the five planes, which is

$$(80 + 4 \cdot 20)/5 = 32,$$

so the average plane was 32% full.

39. You know you have an eighty percent chance of seeing at least one shooting star over the next hour. What is your chance of seeing a shooting star over the next half hour?

Denote by p the probability of observing a shooting star over a half an hour period. The probability of not observing a shooting star over the one hour period is then $(1-p)^2$, which must be equal to 0.2, so we must solve the equation $(1-p)^2 = 1/5$, which gives $p = 1 - 1/\sqrt{5} \approx 0.55$.

40. If the probability of seeing at least one car in an hour is 544/625, what's the probability of seeing no cars in a fifteen minute interval?

Denote by p the probability of observing a car over a fifteen minute interval. The probability of not observing a car over a fifteen minute interval is then $(1-p)^4$, which must be equal to $1 - 544/652 = 81/652$, so we must solve the equation $(1-p)^4 = 81/652 = (3/5)^4$, which gives $p = 1 - 3/5 = 2/5$.